Answers

Chapter 6 Review Packet

Theory/Vocab

1. What is the general form for a confidence interval?

   estimate ± margin of error

2. What is the formula for a confidence interval for a population mean? What part of this formula is the estimate and what part is the margin of error?

   estimate → $\bar{x} \pm \frac{z^* \cdot \sigma}{\sqrt{n}}$ ← margin of error

3. What 3 things can we do to decrease the margin of error in a confidence interval?
   a. increase sample size
   b. decrease σ
   c. choose lower confidence level

   (p. 442)

4. ~~What are some cautions when using confidence intervals?~~
   - we need a SRS
   - outliers affect them greatly
   - must know σ
   - no bias
   - well designed expt.

   (p. 444)

5. If our null hypothesis is Ho: μ = 8.5, give an example of one-sided and two-sided alternative hypotheses.

   One sided: $H_a: \mu > 8.5$ or $H_a: \mu < 8.5$

   Two sided: $H_a: \mu \neq 8.5$

6. What is the formula for the test statistic for a test of significance on a population mean?

   $$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

7. What is the name and distribution for this test statistic?

   z-statistic, standard normal distribution, $N(0,1)$

8. What are the 3 assumptions we make when perform tests of significance? or conf. intervals
   a. SRS
   b. normal pop. or $n \geq 30$
   c. know σ

9. What are the ~~4~~ 5 steps to a test of significance? (p.459-460)
   a) Assump.
   a. state null & alternative hypotheses
   b. Calculate test statistic
   c. calculate P-value
   d. state a conclusion

10. What does statistically significant mean?
    when the p-value is smaller than α, reject Ho
    (see p.458-459 in book)

11. If we want to use a confidence interval to test a two-sided alternative hypothesis, where would the mean we are testing have to fall in order for us to be able to <u>reject our null</u> hypothesis?
    (p.463-466) reject= μ is outside the confidence interval

12. What is power?
    (p.484) The probability that we reject Ho when Ho <u>is</u> false.

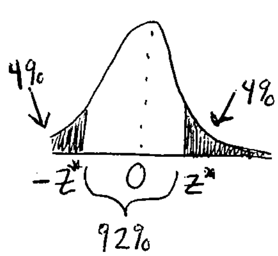
13. What are the 4 things we can do to increase power?
    (p.486)
    a. increase α
    b. consider an alternative μ_A further from μ_0
    c. increase sample size
    d. decrease σ

14. What is Type I error? Give its symbol as well as the definition in a complete sentence
    Prob. of rejecting Ho when Ho is true, α

15. What is Type II error? Give its symbol as well as the definition in a complete sentence
    Prob. of failing to reject Ho when Ho is false, β

<u>Confidence Intervals</u>

16. The average stay for a mother at a local hospital after childbirth is normally distributed with a mean of 2 days and a standard deviation of 0.5 days. An SRS of 15 childbirth mothers produced a sample mean of 1.8 days. Calculate a 92% confidence interval and interpret this interval.
    $\mu = 2 \quad \sigma = 0.5$
    $n = 15 \quad \bar{x} = 1.8$



$$\bar{x} \pm \frac{z^* \cdot \sigma}{\sqrt{n}}$$

$$1.8 \pm \frac{(1.7507)(0.5)}{\sqrt{15}}$$

invnorm(0.04,0,1) = $-1.7507 = -z^*$

$z^* = 1.7507$

(1.574, 2.026)

we are 92% confident that the mean stay ~~for a childbirth mother~~ at a hospital is in (1.574, 2.026)

#16

17. If I wanted to reduce the margin of error in problem [redacted] to 0.2, what would my sample size have to be?

$$n=\left(\frac{z^{*}\cdot\sigma}{m}\right)^2 \quad n=\left(\frac{1.7507\cdot 0.5}{0.2}\right)^2 = 19.1659 \rightarrow \text{round up } 20$$

18. For schools to be eligible for a federal grant, the district's mean income per household must not exceed \$16,000. The school board hired a research firm to gather the required data. Assume that the standard deviation is \$3,000. An SRS of 75 houses produced a mean household income of \$17,000. Calculate a 99% confidence interval and interpret this interval.

$\mu = 16{,}000$
$\bar{x} = 17{,}000$
$\sigma = 3{,}000$
$n = 75$

$$\bar{x} \pm \frac{z^{*}\cdot\sigma}{\sqrt{n}}$$

$$17{,}000 \pm \frac{2.576\cdot 3{,}000}{\sqrt{75}} = (16107.65, 17892.35)$$

We are 99% confident that the mean household income is between \$16107.65 and \$17892.35

Confidence Intervals and Two-Sided Tests

19. During the past several years, frequent checks were made of spending patterns of citizens returning from a vacation of 21 days or less in countries in Europe. Results indicated that travelers spend an average of \$1,010 with a standard deviation of \$300 on various items. An SRS of 270 travelers produces a sample mean of \$1,090. Is there evidence that there has been a change in the average amount of spending? Use a significance level of 0.02, and use your confidence level to state your conclusion in complete sentences.

$\mu = 1{,}010$
$\bar{x} = 1090$
$\sigma = 300$
$n = 270$

$$\boxed{\begin{array}{l} H_0: \mu = 1010 \\ H_a: \mu \neq 1010 \end{array}}$$

$\alpha = 0.02$

Conf. level $= 0.98 \Rightarrow z^{*} = 2.326$

$$\bar{x} \pm \frac{z^{*}\cdot\sigma}{\sqrt{n}}$$

$$(1047.53, 1132.47)$$

We reject H_0 at the $\alpha = 0.02$ level because the mean we are testing (1010) falls outside the 98% conf. interval. We have sufficient evidence to say the average spending is not \$1010.

20. A local manufacturer of doors for home use is attempting to determine if the average height of adult males in this market is has changed from 70 inches. The height of adult males is known to have a standard deviation of 2 inches. A SRS of 121 adult males produced a sample mean of 72 inches. Using a significance level of 0.05, can we conclude that the average height of males has changed? Use a confidence interval to make your conclusion and state this in complete sentences.

$\mu = 70''$
$\bar{x} = 72''$
$\sigma = 2''$
$n = 121$
$\alpha = 0.05$

$95\% \Rightarrow z^{*} = 1.96$

$$\boxed{\begin{array}{l} H_0: \mu = 70'' \\ H_a: \mu \neq 70'' \end{array}}$$

$$\bar{x} \pm \frac{z^{*}\cdot\sigma}{\sqrt{n}}$$

$$(71.644, 72.356)$$

We reject H_0 at the $\alpha = 0.05$ level because the mean we are testing (70) is outside our 95% confidence level. We have sufficient evidence to say that the mean height of adult males in this market has changed from 70".

Assumptions
1. σ known
2. SRS
3. normal pop or $n \geq 30$

AP Statistics- Tests of Significance

- **Assumptions**
- **Hypotheses**
- **Test statistic**
- **P-value**
- **Conclusion**

21) A manufacturer claims that a new type of showerhead uses less water than the last model. It is known that the last model used on average 23.6 gallons of water for a 20-minute shower. A consumer agency runs a test on a SRS of size 50. If the sample mean is 22.8 gallons and we know the population standard deviation is 2.6 gallons, should the manufacturer's claim be rejected at a significance level of 5%? Of 1%?

Assumptions
1. ✓
2. ✓
3. $n = 50 \geq 100$

$H_0: \mu = 23.6$

$H_a: \mu < 23.6$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = -2.1757$

$P(z < -2.1757) = 0.0148$

reject @ 5% level
fail to reject @ 1% level

22) A 2004 study reports that the mean amount of money that a 4-person family spends eating out (in restaurants, at fast food places, at Wawa, etc.) in a week is \$202.56. An economist believes this has gone up. He uses a SRS of 135 families, and finds a mean of \$203.41 with a standard deviation of \$4.96. Should the claim be rejected at a 1% level of significance?

Assumptions
1. ✓
2. ✓
$n = 135 \geq 30$ ✓

$H_0: \mu = 202.56$

$H_a: \mu > 202.56$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = 1.991$

$P(z > 1.991) = 0.0232$

$\alpha = 0.01 \Rightarrow$ fail to reject

23) As calculators have gotten more advanced, so have their prices. Certain prices are lower, but some are higher. In 2000 (when Miss Senske graduated from high school!) the average price of TI calculators was \$95.34. You believe that the average proce has changed. You test the claim by looking at the prices of a SRS of 70 TI calculators and find a mean of \$93.87. Assuming the standard deviation is \$9.75, is there evidence to reject the claim?

Assumptions
1. ✓
2. ✓
3. $n = 70 \geq 30$ ✓

$H_0: \mu = 95.34$

$H_a: \mu \neq 95.34$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = -1.26$

$2 \cdot P(z < -1.26) = 0.2072$

$\alpha = 0.05 \Rightarrow$ fail to reject

24) Central Bucks School District reports that the mean number of days absent during the school year for 12th grade students is 10.4 and the standard deviation is 2.8. Many seniors think that this is not true. The senior students look at the attendance records of an SRS of 52 senior students and find the mean number of days absent to be 9.3. Is this evidence that the CBSD claim is incorrect?

Assumptions
1. ✓
✓
3. $n = 52 \geq 30$ ✓

$H_0: \mu = 10.4$

$H_a: \mu \neq 10.4$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = -2.833$

$2 \cdot P(z < -2.833) = 0.0046$

$\alpha = 0.05 \Rightarrow$ reject

Power and Error

25. A teacher performs a test of significance to determine if the mean score on a final exam differs from 75 (percent). The test is done with a significance level of 0.05. The population standard deviation is 12, and the sample size is 100. Will the test adequately detect if the mean score has changed to 77?

    a. State the hypotheses, the alternative that we think, the significance level, and all other important information.

    $H_0: \mu = 75$ $\quad \alpha = 0.05$ $\quad \mu_A = 77$

    $H_a: \mu \neq 75$ $\quad \sigma = 12$

    $n = 100$

    b. We are told the power of this test of significance is 0.886. Is this test of significance good enough to detect the stated change in the mean?

    yes, power > 0.80

    c. Find the probability of a Type I error:

    $\alpha = 0.05$

    d. Find the probability of a Type II error:

    $\beta = 1 - 0.886 = 0.114$

    e. What would happen to the power if we used an alternative of 74? Would it increase or decrease?

    decrease (74 is closer to 75 than 77 was)

    f. What would happen to the power if we increased the sample size to 150? Would it increase or decrease?

    new n, old n

    increase (150 > 100)

    g. What would happen to the power if we realized that the population standard deviation is actually 15? Would it increase or decrease?

    new σ, old σ

    decrease (15 > 12, so data is more spread out)

    h. What would happen to the power if we used a significance level of 0.07?

    increase ($\alpha \uparrow$, so more chance of rejecting H_0)